

Nucleation at Finite Temperature Beyond the Superminispace Model

J.-Q. Liang ^{1,2*}, H. J. W. Müller-Kirsten ^{1†}, D. K. Park ^{1,3‡} and A. V. Shurgaila ^{1§}

1. Department of Physics, University of Kaiserslautern, D-67653 Kaiserslautern, Germany

2. Department of Physics, and Institute of Theoretical Physics, Shanxi University, Taiyuan, Shanxi, 030006, P.R. China

3. Department of Physics, Kyungnam University, Masan, 631-701, Korea

Abstract

The transition from the quantum to the classical regime of the nucleation of the closed Robertson-Walker Universe with spacially homogeneous matter fields is investigated with a perturbation expansion around the sphaleron configuration. A criterion is derived for the occurrence of a first-order type transition, and the related phase diagram for scalar and vector fields is obtained. For scalar fields both the first and second order transitions can occur depending on the shape of the potential barrier. For a vector field, here that of an $O(3)$ nonlinear σ -model, the transition is seen to be only of the first order.

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*email: jqliang@mail.sxu.edu.cn

†email: mueller1@physik.uni-kl.de

‡email: dkpark@genphys.kyungnam.ac.kr

§email: shurgaila@physik.uni-kl.de

I. INTRODUCTION

The problem of the decay rate of a metastable state and coherence of degenerate states via quantum tunneling has profound physical implications in many fundamental phenomena in various branches of physics as e.g. in condensed matter and particle physics and cosmology. The instanton techniques initiated long ago by Langer [1] and Coleman [2] are a major tool of investigation, and provide us with a formalism capable of producing accurate values for the tunneling rate. The instanton method has been widely used to study tunneling at zero temperature. The generalization to finite temperature tunneling has been a long-standing problem in which a new type of solution satisfying a periodic boundary condition, and therefore called the periodic instanton, was gradually realized to be relevant [3,4]. The exact analytic form of the periodic instanton is known only in one-dimensional quantum mechanics [5]. In field theory models, it can be found either approximately at low energies or numerically. Thus quantum tunneling at finite temperature T is, under certain conditions, dominated by periodic instantons with finite energy E , and in the semi-classical approximation the euclidean action is expected to be saturated by a single periodic instanton. Thus only periodic instantons with the period equal to the inverse temperature can dominate the thermal rate. With exponential accuracy the tunneling probability $P(E)$ at a given energy E can be written as

$$P(E) \sim e^{-W(E)} = e^{-S(\beta)+E\beta} \quad (1)$$

The period β of the periodic instanton is related to the energy E in the standard way $E = \frac{\partial S}{\partial \beta}$ and $S(\beta)$ is the action of the periodic instanton per period. With increasing temperature thermal hopping becomes more and more important and beyond some critical or crossover temperature T_c becomes the decisive mechanism. The barrier penetration process is then governed by a static solution of the euclidean field equation, i.e. the sphaleron. The study of the crossover from quantum tunneling to thermal hopping is an interesting subject with a long history [6,7]. Under certain assumptions for the shape of the potential barrier, it

was found that the transition between quantum tunneling and thermally assisted hopping occurs at the temperature T_c and was recognized as a smooth second order transition in the quantum mechanical models of Affleck [6] and the cosmological models of Linde [7]. It was demonstrated that the periodic instantons which govern the tunneling in the intermediate temperature regime interpolate smoothly between the zero temperature or vacuum instanton and the sphaleron. In analogy with the terminology of statistical mechanics this phenomenon can be referred to as a second-order transition characterized by the plot of euclidean action $S(\beta)$ versus instanton period β , the latter being the inverse temperature in the finite temperature field theory.

However, it was shown [8] that the smooth transition is not generic. Using a simple quantum mechanical model it was demonstrated that the time derivative of the euclidean action would be discontinuous if the period of the instanton is not a monotonic function of energy. Assuming that there exists a minimum of $\beta(E)$, i. e. that $\frac{d\beta}{dE} = 0$ at some value of E , the second time derivative of the action

$$\frac{d^2 S(\beta)}{d\beta^2} = \frac{1}{\frac{d\beta}{dE}} \quad (2)$$

would not be defined at the minimum, or, in other words, the first time derivative is discontinuous. The sharp first order transition occurs as a bifurcation in the plot of the action $S(\beta)$ versus the period β . In the context of field theory the crossover behaviour and the bifurcation of periodic instantons have also been explained in a more transparent manner [9]. The idea to determine the order of a transition from the plot of euclidean action versus the period of the instantons was subsequently extended, and a sufficient condition for the existence of a first-order transition was derived using only small fluctuations around the sphaleron. If the period $\beta(E \rightarrow U_0)$ of the periodic instanton close to the barrier peak can be found, a sufficient condition to have a first-order transition is seen to be that $\beta(E \rightarrow U_0) - \beta_s < 0$ or $\omega^2 - \omega_s^2 > 0$, where U_0 denotes the barrier height and β_s is the period of small oscillation around the sphaleron [10,11]; ω and ω_s are the corresponding frequencies. This observation triggered active research on the transition behaviour, as e.g. in connection with spin tunnel-

ing in condensed matter physics [12–14] and with tunneling in various field theory models [15–19].

It is also interesting to investigate the crossover from quantum tunneling to thermal hopping in the context of cosmology. After the pioneering attempt of Linde [7] little work has been done along this direction in view of the lack of solvable models. Motivated by a similar study of bubble nucleation in field theory [18,19], the nucleation rate in a superminispace model [20] has been extended to finite temperature [21] where the matter field is frozen out and leaves only the constant vacuum energy density. In this context a periodic instanton solution was obtained analytically, and the transition from tunneling to thermal hopping was found to be of the first order. The superminispace model may be too simple to imply a realistic result. In the present paper we therefore extend the model to one including a spacially homogeneous matter field. Small fluctuations around the sphaleron solutions are then studied with a perturbation method. Criteria for a first-order transition and related phase diagrams are obtained for both scalar and vector fields. This investigation may shed light on our understanding of the time evolution of the early Universe. In Sec. 2 the effective Lagrangian and the equations of motion are obtained for the closed Robertson–Walker (RW) metric interacting with spacially homogeneous scalar and vector fields. The physical meaning of tunneling is explained. The oscillation frequencies around the sphaleron are obtained with a perturbation expansion in Sec. 3. We derive the criterion for the first-order transition of the nucleation and the related phase diagram for interaction with a scalar field. In Sec. 4 we apply a similar approach to the model with a vector field.

II. SPHALERONS AND THE THERMAL RATE OF NUCLEATION

We begin with the model of the Universe defined by the action,

$$S = \int d^4x \sqrt{-g} \left[-\frac{\mathcal{R}}{16\pi G} + \mathcal{L}_m \right] \quad (3)$$

where \mathcal{R} is the Ricci scalar. The Lagrangian density of the scalar matter field ϕ is of the general form,

$$\mathcal{L}_m = \frac{1}{G_\phi} \left[\frac{1}{2} \partial_\mu \phi \partial^\mu \phi - V(\phi) \right] \quad (4)$$

where G_ϕ is a dimensional parameter. For a vector field we consider that of the $O(3)$ nonlinear σ -model with a symmetry breaking term such as,

$$\mathcal{L}_m = \frac{1}{2} m \sum_a \partial_\mu n_a \partial^\mu n_a - \frac{1}{\lambda^2} (1 + n_3), \quad \sum_{a=1}^3 n_a^2 = 1 \quad (5)$$

where m and λ are suitable parameters. Contemporary cosmological models are based on the idea that the Universe is pretty much the same everywhere. More mathematically precise properties of the manifold may be isotropy and homogeneity. The spacetime to be considered here is $\mathbf{R} \times \Sigma$ where \mathbf{R} represents the time direction and Σ is a homogeneous and isotropic 3-manifold. The Universe is also assumed to be closed. We therefore obtain the Robertson–Walker (RW) metric of the closed case,

$$ds^2 = dt^2 - R^2(t) d\Omega_3^2 \quad (6)$$

The manifold Σ in our case is a three-sphere S^3 and the lapse function is simply equal to 1. $R(t)$ is known as the scale factor which tells us “how big” the spacetime slice Σ is at time t . $d\Omega_3^2$ is the metric on a unit 3-sphere. The Ricci scalar is found to be

$$\mathcal{R} = -6 \left[\frac{\ddot{R}}{R} + \frac{\dot{R}^2}{R^2} + \frac{1}{R^2} \right] \quad (7)$$

where a dot denotes the time derivative. For spatially homogeneous matter fields $\phi = \phi(t)$ and $\mathbf{n} = \mathbf{n}(t)$ the angle integrals can be carried out and we have,

$$S = \int L dt \quad (8)$$

The effective Lagrangians are obtained as

$$L = 2\pi^2 \left\{ -\frac{3R(\dot{R}^2 - 1)}{8\pi G} + \frac{1}{G_\phi} \left[\frac{1}{2} R^3 \dot{\phi}^2 - R^3 V(\phi) \right] \right\} \quad (9)$$

for the scalar field and

$$L = 2\pi^2 \left[-\frac{3R(\dot{R}^2 - 1)}{8\pi G} + \frac{mR^3}{2} \sum_a \dot{n}_a^2 - \frac{R^3}{\lambda^2} (1 + n_3) \right] \quad (10)$$

for the vector field. The canonical momenta are defined by

$$P_R = \frac{\partial L}{\partial \dot{R}} = -\frac{3\pi R \dot{R}}{2G}, \quad P_\phi = \frac{\partial L}{\partial \dot{\phi}} = \frac{2\pi^2}{G_\phi} R^3 \dot{\phi}, \quad P_a = 2\pi^2 m R^3 \dot{n}_a \quad (11)$$

The corresponding Hamiltonians

$$H = \frac{G_\phi}{4\pi^2 R^3} P_\phi^2 - \frac{G}{3\pi R} P_R^2 - \frac{3\pi}{4G} R + \frac{2\pi^2}{G_\phi} R^3 V(\phi), \quad (12)$$

$$H = \frac{1}{4\pi^2 m R^3} P_a^2 - \frac{G}{3\pi R} P_R^2 - \frac{3\pi}{4G} R + \frac{2\pi^2}{\lambda^2} R^3 (1 + n_3) \quad (13)$$

are conserved quantities. For our purposes of the study of nucleation we make use of the Wick rotation $\tau = it$ and obtain the euclidean Lagrangians,

$$L_e = 2\pi^2 \left\{ -\frac{3R(\dot{R}^2 + 1)}{8\pi G} + \frac{1}{G_\phi} \left[\frac{1}{2} R^3 \dot{\phi}^2 + R^3 V(\phi) \right] \right\}, \quad (14)$$

$$L_e = 2\pi^2 \left[-\frac{3R(\dot{R}^2 + 1)}{8\pi G} + \frac{m}{2} R^3 \sum_a \dot{n}_a^2 + \frac{R^3}{\lambda^2} (1 + n_3) \right] \quad (15)$$

From now on the dot denotes imaginary time derivatives, e. g. $\dot{R} = \frac{dR}{d\tau}$.

The euclidean equations of motion for the scalar field are seen to be

$$\frac{d}{d\tau}(R\dot{R}) - \frac{\dot{R}^2 + 1}{2} + 2\pi\tilde{G}R^3\dot{\phi}^2 + 4\pi\tilde{G}R^2V(\phi) = 0 \quad (16)$$

where $\tilde{G} = \frac{G}{G_\phi}$, and

$$\frac{d}{d\tau}(R^3\dot{\phi}) = R^3 \frac{\partial V}{\partial \phi} \quad (17)$$

The sphalerons ϕ_0 and R_0 are static solutions of eqs. (16) and (17) with $\dot{\phi} = \ddot{\phi} = \dot{R} = \ddot{R} = 0$.

From eq.(16) we have

$$R_0 = \left[\frac{1}{8\pi\tilde{G}V(\phi_0)} \right]^{\frac{1}{2}} \quad (18)$$

ϕ_0 is the position of the peak of the potential barrier such that $\frac{\partial V}{\partial \phi}|_{\phi=\phi_0} = 0$. With the sphaleron ϕ_0 the effective potential of the dynamical variable R is seen to be from eq.(9),

$$U(R) = -\frac{R^3 V(\phi_0)}{G_\phi} + \frac{3R}{8\pi G} \quad (19)$$

R_0 is just the position of the above potential barrier peak and indeed the sphaleron. The thermal rate of nucleation at temperature T is given by the Arrhenius law,

$$P(T) \sim e^{-\frac{U(R_0)}{T}} \quad (20)$$

Our superminispace model here is simply the dynamical model described by the equation of motion (16) with the scalar field variable ϕ in $V(\phi)$ replaced by the sphaleron ϕ_0 . The nucleation process in the superminispace model has been extended to the finite temperature case with the periodic instanton formalism in our previous work [21]. In the present paper the scalar field is not frozen out and we instead investigate the fluctuation of the fields around the sphalerons. The crossover behaviour from tunneling to thermal hopping can be obtained with perturbation expansions.

III. NUCLEATION AT FINITE TEMPERATURE IN PRESENCE OF A SCALAR FIELD

As we demonstrated above, the crossover behaviour of the nucleation of our model Universe from quantum tunneling to thermal activation can be obtained from the deviation of the period of the periodic instanton from that of the sphaleron. To this end we expand the field variables about the sphaleron configurations ϕ_0 and R_0 , i. e. we set

$$\phi = \phi_0 + \xi, \quad R = R_0 + \eta \quad (21)$$

where ξ and η are small fluctuations. Substitution of eq.(21) into the equations of motion (16) and (17) yields the following power series equations of the fluctuation fields ξ and η ,

$$\hat{l} \begin{pmatrix} \xi \\ \eta \end{pmatrix} = \hat{h} \begin{pmatrix} \xi \\ \eta \end{pmatrix} + \begin{pmatrix} G_2^\xi(\xi, \eta) \\ G_2^\eta(\xi, \eta) \end{pmatrix} + \begin{pmatrix} G_3^\xi(\xi, \eta) \\ G_3^\eta(\xi, \eta) \end{pmatrix} + \begin{pmatrix} G_4^\xi(\xi, \eta) \\ G_4^\eta(\xi, \eta) \end{pmatrix} + \dots \quad (22)$$

where the operators \hat{l}, \hat{h} are defined as

$$\hat{l} = \begin{pmatrix} \frac{d^2}{d\tau^2} & 0 \\ o & \frac{d^2}{d\tau^2} \end{pmatrix}, \quad \hat{h} = \begin{pmatrix} V^{(2)}(\phi_0) & 0 \\ o & -8\pi\tilde{G}V(\phi_0) \end{pmatrix} \quad (23)$$

and G_2, G_3, \dots denote terms which contain quadratic, cubic and higher powers of the small fluctuations respectively:

$$\begin{aligned} G_2^\xi &= -\frac{3}{R_0} [\dot{\eta}\dot{\xi} + \eta\ddot{\xi}] + \frac{1}{2}V^{(3)}(\phi_0)\xi^2 + \frac{3}{R_0}V^{(2)}(\phi_0)\xi\eta, \\ G_3^\xi &= -\frac{6}{R_0^2}\eta\dot{\eta}\dot{\xi} - \frac{3}{R_0^2}\eta^2\ddot{\xi} + \frac{1}{3!}V^{(4)}(\phi_0)\xi^3 + \frac{3}{2R_0}V^{(3)}(\phi_0)\eta\xi^2 + \frac{3}{R_0^2}V^{(2)}(\phi_0)\xi\eta^2, \\ G_4^\xi &= -\frac{3}{R_0^3}\eta^2\dot{\eta}\dot{\xi} - \frac{1}{R_0^3}\eta^3\ddot{\xi} + \frac{1}{4!}V^{(5)}(\phi_0)\xi^4 + \frac{1}{2R_0}V^{(4)}(\phi_0)\xi^3\eta + \frac{3}{2R_0^2}V^{(3)}(\phi_0)\eta^2\xi^2 + \frac{1}{R_0^3}V^{(2)}(\phi_0)\xi\eta^3, \\ G_2^\eta &= -\frac{1}{2R_0}\dot{\eta}^2 - \frac{1}{R_0}\eta\ddot{\eta} - 2\pi\tilde{G}R_0\xi^2 - 2\pi\tilde{G}R_0V^{(2)}(\phi_0)\xi^2 - \frac{4\pi\tilde{G}}{R_0}V(\phi_0)\eta^2, \\ G_3^\eta &= -4\pi\tilde{G}\eta\xi^2 - 4\pi\tilde{G}V^{(2)}(\phi_0)\eta\xi^2 - \frac{4\pi\tilde{G}}{3!}R_0V^{(3)}(\phi_0)\xi^3, \\ G_4^\eta &= -\frac{2\pi\tilde{G}}{R_0}\eta^2\xi^2 - \frac{8\pi\tilde{G}}{3!}V^{(3)}(\phi_0)\eta\xi^3 - \frac{2\pi\tilde{G}}{R_0}V^{(2)}(\phi_0)\xi^2\eta^2 - \frac{\pi\tilde{G}R_0}{3!}V^{(4)}(\phi_0)\xi^4 \end{aligned}$$

where $V^{(n)}(\phi_0) = \frac{d^n V(\phi)}{d\phi^n}|_{\phi=\phi_0}$. The first-order solution of the fluctuation fields is obvious from eq.(22). We have

$$\xi \sim \cos \omega_0 \tau, \quad \eta \sim \cos \omega_0 \tau, \quad \omega_0^2 = \frac{1}{R_0^2} = -V^{(2)}(\phi_0) \quad (24)$$

where ω_0 is the frequency of the sphalerons which is simply the frequency of small oscillations in the bottoms of the inverted potential wells of $U(R)$ and $V(\phi)$. Substituting the first-order solution into eq.(22) we obtain the second-order solution; the higher-order results can be obtained by successive substitutions. After the second substitution we have,

$$\xi = \rho \cos \omega \tau + \rho^2 [g_{1,\xi} + g_{2,\xi} \cos 2\omega \tau] + \xi_3, \quad (25)$$

$$\eta = \rho \cos \omega \tau + \rho^2 g_{2,\eta} \cos 2\omega \tau + \eta_3, \quad (26)$$

where

$$\begin{aligned} g_{1,\xi} &= \frac{1}{2\omega_0^2} \left[\frac{1}{2}V^{(3)}(\phi_0) - 3\omega_0^3 \right], \\ g_{2,\xi} &= -\frac{1}{6\omega_0^2} \left[3\omega_0^3 + \frac{1}{2}V^{(3)}(\phi_0) \right], \\ g_{2,\eta} &= -\frac{1}{3\omega_0} \left[\frac{3}{4}\omega_0^2 + 2\pi\tilde{G}(1 - V(\phi_0)) \right]. \end{aligned}$$

In our case $g_{1,\eta} = 0$. Here ρ serves as a perturbation parameter. The third-order corrections ξ_3, η_3 are proportional to ρ^3 . Substitution of eqs.(25), (26) into the equation of motion (22) yields an equation to determine ξ_3, η_3 and the corresponding frequency ω . After some tedious algebra we obtain the deviation of the frequency from ω_0 up to order of ρ^4 , i.e.

$$\omega^2 - \omega_0^2 = -\rho^2 \frac{4\pi\tilde{G}}{3\omega_0^2} V^{(3)}(\phi_0) g_{1,\xi} - \rho^4 2\pi\tilde{G} \left[\frac{V^{(4)}(\phi_0)}{3\omega_0^2} + 2\omega_0^2 \right] g_{1,\xi}^2 \quad (27)$$

The ρ^4 term applies in case the ρ^2 term vanishes. The sufficient condition for a transition of the first order to occur is $\omega^2 - \omega_0^2 > 0$. In Fig. 2 we show the phase diagram taking into account terms up to the order of ρ^2 .

We now analyse some field models in terms of our criterion eq.(27). For the well studied ϕ^4 model,

$$V(\phi) = (\phi^2 - \alpha^2)^2 \quad (28)$$

we have $\phi_0 = 0$, $\omega_0^2 = -V(\phi_0) = 4\alpha^2$, $V^{(3)}(\phi_0) = 0$ and $V^{(4)}(\phi_0) = 24$. Eq.(27) leads to

$$\omega^2 - \omega_0^2 < 0 \quad (29)$$

The transition is of second order, in agreement with previous observations in the literature [6,7]. In recent investigations it was pointed out that the transition can be first order with a steeper well of the potential [9],

$$V(\phi) = \frac{4+\alpha}{12} - \frac{1}{2}\phi^2 - \frac{\alpha}{4}\phi^4 + \frac{1+\alpha}{6}\phi^6 \quad (30)$$

which is a double-well type for $\alpha > 0$ (see Fig. 3). The sphaleron is $\phi_0 = 0$ with frequency $\omega_0 = 1$. Since $V^{(3)}(\phi_0) = 0$ the criterion for the first-order transition is determined by the ρ^4 term such that

$$\omega^2 - \omega_0^2 = -18\pi\rho^4[1 - \alpha]\tilde{G} \quad (31)$$

We thus have either first or second order transitions depending on the parameter α . When $0 < \alpha < 1$ the transition is of second order, while for $\alpha > 1$ it is of the first order.

IV. VECTOR MATTER FIELD

The winding number transition at finite temperature, i. e. the transition between degenerate vacua with the vector field model of eq.(5), has been investigated recently using a similar method, but in a flat space-time [16,17]. It was found that in that context the transition is always of the first order. We now consider the nucleation of the model Universe in the presence of the same vector field. We reexpress the vector field with unit norm in terms of angular variables, i.e.

$$\mathbf{n} = (\sin \theta \cos \varphi, \sin \theta \sin \varphi, \cos \theta) \quad (32)$$

The euclidean equations of motion are found from the Lagrangian (15) to be

$$\frac{d}{d\tau}(R^3\dot{\theta}) - R^3 \sin \theta \cos \theta \dot{\varphi} + \frac{R^3}{m\lambda^2} \sin \theta = 0, \quad (33)$$

$$\frac{d}{d\tau}(R\dot{R}) - \frac{1 + \dot{R}^2}{2} + 2\pi G R^2 \left[m \sum_a \dot{n}_a^2 + \frac{2}{\lambda^2}(1 + \cos \theta) \right] = 0, \quad (34)$$

$$\frac{d}{d\tau}(R^3\dot{\varphi} \sin^2 \theta) = 0 \quad (35)$$

where

$$\dot{n}_1 = \dot{\theta} \cos \theta \cos \varphi - \dot{\varphi} \sin \theta \sin \varphi,$$

$$\dot{n}_2 = \dot{\theta} \cos \theta \sin \varphi + \dot{\varphi} \sin \theta \cos \varphi,$$

$$\dot{n}_3 = -\dot{\theta} \sin \theta.$$

The sphaleron solution which is obtained from $\dot{\theta} = \ddot{\theta} = \dot{\varphi} = \ddot{\varphi} = \dot{R} = \ddot{R} = 0$ is seen to be

$$\mathbf{n}_0 = (0, 0, 1), \quad R_0 = \frac{\lambda}{4\sqrt{\pi G}} \quad (36)$$

with $\theta_0 = 0$ and φ_0 an arbitrary constant. We again consider the perturbation expansion around the sphaleron configurations and set

$$\theta = \theta_0 + \gamma, \quad \varphi = \varphi_0 + \delta, \quad R = R_0 + \zeta \quad (37)$$

A self consistent solution is determined by

$$\hat{l} \begin{pmatrix} \gamma \\ \zeta \end{pmatrix} = \hat{h}_v \begin{pmatrix} \gamma \\ \zeta \end{pmatrix} + \begin{pmatrix} G_2^\gamma(\gamma, \zeta) \\ G_2^\zeta(\gamma, \zeta) \end{pmatrix} + \begin{pmatrix} G_3^\gamma(\gamma, \zeta) \\ G_3^\zeta(\gamma, \zeta) \end{pmatrix} + \begin{pmatrix} G_4^\gamma(\gamma, \zeta) \\ G_4^\zeta(\gamma, \zeta) \end{pmatrix} + \dots \quad (38)$$

with $\delta = \text{const.}$ where

$$\hat{h}_v = \begin{pmatrix} -\frac{1}{m\lambda^2} & 0 \\ o & -\frac{16\pi G}{\lambda^2} \end{pmatrix} \quad (39)$$

and

$$\begin{aligned} G_2^\gamma &= -\frac{3}{R_0} \left[\dot{\zeta}\dot{\gamma} + \zeta\ddot{\gamma} + \frac{1}{m\lambda^2}\zeta\gamma \right], \\ G_3^\gamma &= -\frac{6}{R_0^2}\zeta\dot{\zeta}\dot{\gamma} - \frac{3}{R_0^2}\zeta^2\ddot{\gamma} + \frac{1}{3!m\lambda^2}\gamma^3 + \gamma\delta^2 - \frac{3}{R_0^2m\lambda^2}\gamma\zeta^2, \\ G_4^\gamma &= -\frac{3}{R_0^3}\zeta^2\dot{\zeta}\dot{\gamma} - \frac{1}{R_0^3}\zeta^3\ddot{\gamma} + \frac{3}{R_0}\zeta\delta^2\gamma + \frac{1}{2m\lambda^2R_0}\zeta\gamma^3 - \frac{1}{m\lambda^2R_0^3}\zeta^3\gamma, \\ G_2^\zeta &= -2\pi GmR_0\dot{\gamma}^2 - \frac{1}{R_0}\zeta\ddot{\gamma} + \frac{2\pi G}{\lambda^2}R_0\gamma^2 - \frac{8\pi G}{\lambda^2R_0}\zeta^2 - \frac{1}{2R_0}\dot{\zeta}^2, \\ G_3^\zeta &= -4\pi Gm\zeta\dot{\gamma}^2 + \frac{4\pi G}{\lambda^2}\zeta\gamma^2, \\ G_4^\zeta &= -2\pi mGR_0\gamma^2\dot{\delta}^2 - \frac{2\pi Gm}{R_0}\zeta^2\dot{\gamma}^2 - \frac{\pi G}{3!\lambda^2}R_0\gamma^4 + \frac{2\pi G}{\lambda^2R_0}\zeta^2\gamma^2. \end{aligned}$$

The solution for the fluctuation in first-order approximation is

$$\gamma \sim \cos \omega_0 \tau, \quad \zeta \sim \cos \omega_0 \tau \quad (40)$$

where the frequency of the sphaleron is found to be

$$\omega_0 = \frac{4\sqrt{\pi G}}{\lambda} = \frac{1}{R_0}, \quad m = \frac{1}{16\pi G} \quad (41)$$

The solution for fluctuations up to the third-order approximation is

$$\gamma = \rho \cos \omega \tau - \frac{\rho^2}{2}[3\omega_0 + \omega_0 \cos 2\omega \tau] + \gamma_3, \quad (42)$$

$$\zeta = \rho \cos \omega \tau - \frac{\rho^2}{6\omega_0} \left(\frac{1}{4} + \omega_0^2 \right) \cos 2\omega \tau + \zeta_3, \quad (43)$$

where γ_3 , ζ_3 and ω are determined by substitution of eqs.(42), (43) into the equation of motion (38). By doing so the deviation of the frequency which we are interested in is obtained as,

$$\omega^2 - \omega_0^2 = \rho^2 \frac{3\omega_0^2}{2}(1 + 3\omega_0^2) > 0 \quad (44)$$

which is always positive. The transition is therefore of first order, and is, remarkably, the same as that of the winding number transition of the vector field model in a flat space-time.

V. CONCLUSION

We believe that the present study is the first attempt to investigate the nucleation of a RW closed Universe at finite temperature with time-dependent matter fields. Although we consider only the crossover behaviour of the nucleation from quantum tunneling to thermal activation, this investigation may shed light on the understanding of the time-evolution of the early Universe. Unlike the superminispace model in which only the first order transition exists, we find that both first and second order transitions are possible here, depending on the shape of the potential of the matter fields. From another point of view the system considered here can be regarded as the barrier penetration of the field models in the closed RW metric. A remarkable observation is that the crossover behaviours, i.e. (1)the second order transition for the ordinary ϕ^4 double-well potential of the scalar field, (2) both the first and second for a steeper double-well potential, but (3) first order only for the $O(3)$ nonlinear σ model, maintain the same relations as those of transitions of these field models in a flat space-time.

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Figure Captions

Fig. 1: Barrier of nucleation and the sphaleron R_0

Fig. 2: Phase diagram with scalar field. I. first order region. II. second order region.

Fig. 3: Steeper double-well potentials with $\alpha = 0.1$ and 4